Sr. No. 7676

Exam Code: 211004
Subject Code: 4288

M.Sc(Mathematics) - 4th Semester

(2519)

Paper: Math-581

Functional Analysis-II

Time allowed: 3 hrs. Max. Marks: 100

Note : **Attempt two questions from each unit. Each question carries equal marks.**

Unit I

Q.1 Define weak convergence in a normed linear space. Let (x_n) be a weakly convergent sequence in a normed space X, say, $x_n \rightarrow^w x$. Then prove that

(a) The weak limit x of (x_n) is unique

(b) The sequence $(\|x_n\|)$ is bounded.

Q.2 Let (x_n) be a sequence in a normed space X. Then prove that

(a) Strong convergence implies weak convergence with the same limit.

(b) The converse of (a) is not generally true. Justify your answer.

Q.3 Let A be a set in a normed space X such that every non empty subset of A contains a weak cauchy sequence. Then show that A is bounded.

Q.4 Prove that in a Hilbert space H, $x_n \rightarrow^w x$ if and only if $< x_n, z > \rightarrow < x, z >$ for all $z \in H$.

Unit II

Q.5 Let $T : H \rightarrow H$ be a bounded linear operator on a Hilbert space H. Then prove that

(a) If T is self-adjoint, then $< Tx, x >$ is real for all $x \in H$

(b) If H is complex Hilbert space and $< Tx, x >$ is real for all $x \in H$, then prove that the operator T is self adjoint.

Q.6 Prove that a bounded linear operator T on a complex Hilbert space H is unitary iff T is isometric and surjective.

Q.7 Prove that the product of two bounded self-adjoint linear operator S and T on a Hilbert space H is self adjoint iff the operator commutes.

PTO

Q.8 Let the operator $U : H \to H$ and $V : H \to H$ be unitary where H is a Hilbert space.Then prove that

(a) U is isometric, thus $\|Ux\| = \|x\| \; \forall x \in H$.

(b) UV is unitary.

(c) U is normal.

Unit III

Q.9 Prove that a linear operator on a finite dimensional complex normed space $0 \neq X$ has atleast one eigen value.

Q.10 State and prove Spectral theorem for normal operators.

Q.11 Prove that the spectrum $\sigma(T)$ of a bounded linear operator $T : X \to X$ on a complex Banach space X is compact.

Q.12 Proved that the spectrum $\sigma(T)$ of a bounded linear operator $T : X \to X$ on a complex Banach space X is closed.

Unit IV

Q.13 Define compact linear operator. Let X and Y be normed spaces. Then prove that

(a) Every compact linear operator $T : X \to Y$ is bounded, and hence continuous.

(b) If $dimX = \infty$, then the identity operator $I : X \to X$ is not compact.

Q.14 Prove compactness of $T : l^2 \to l^2$ defined by $y = (e_j) = Tx$ where $e_j = \frac{\zeta_j}{j}$ for $j = 1, 2, 3 \cdots$

Contd... Pg.3

Q.15 Let X and Y be normed spaces and $T : X \to Y$ be a linear operator. Then T is compact iff it maps every bounded sequence (x_n) in X onto a sequence $(T(x_n))$in Y which has a convergent subsequence.

Q.16 Let $T : X \to X$ be a compact linear operator and $S : X \to X$ be a bounded linear operator. Then ST and TS are compact.

Unit V

Q.17 Define Banach algebra and give two examples of a commutative Banach algebra. Justify your answer.

Q.18 Let A be a complex Banach algebra with identity e. If $x \in A$ satisfies $\|x\| < 1$, then $e - x$ is invertible and $(e - x)^{-1} = e + \sum_{j=1}^{\infty} x^j$

Q.19 Let X be a Banach algebra and G denote the set of invertible elements of X. Then

(a) G is a group under multiplication.

(b) G is an open subset of X.

(c) The map $x \mid\to x$ is continuous.

Q.20 Derive the formula for the spectral radius.

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